ML Antenna Pattern Shape and Pointing Estimation in Synthetic Aperture Radar

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Abstract. Radiometric calibration is a fundamental item in the processing of Synthetic Aperture Radar (SAR) data and their exploitation since it allows accurate measures of radar reflectivity. The determination of an accurate azimuth antenna pattern (AAP) and pointing acquires particular relevance for a calibration process. AAP estimation has been traditionally performed using transponders that are high precision and geolocated devices able to provide, when illuminated by a SAR antenna, a fixed RCS with high accuracy. The use of transponders is however difficult and expensive since they require maintenance and accurate and complex calibration. The previous objections justify the search of methods for estimating the AAP using natural targets. The natural targets the research wants to exploit are: sparse nearly over all the acquired images, dense, i.e. many targets can be present in a single image, making the estimation robust; stable in time. In this paper a maximum likelihood estimate of the AAP shape and pointing is proposed, which is based on a multi-channel model (more targets in more images). Pointing is particularly relevant, since an accurate Doppler parameter estimation allows a better collocation of the targets and SNR of the image. Accuracy of the method, especially in Doppler centroid estimation (orientation of antenna maximum), is shown to be better than for traditional, correlation-based methods. Experimental results based both on simulation and Cosmo SkyMed stripmap dataset are presented.

Keywords. Synthetic Aperture Radar, Persistent Scatterer, Azimuth Antenna Pattern Estimation, Doppler Centroid Estimation, Maximum Likelihood.

1. Introduction

Radiometric calibration is a fundamental item in the processing of Synthetic Aperture Radar (SAR) data and their exploitation since it allows accurate measures of radar reflectivity. The use of repeat pass images can monitor the overall gain variation with time (transmit and receive gains, propagation and processor gain). The determination of an accurate azimuth antenna pattern (AAP) and attitude, i.e. accurate Doppler centroid, acquires hence particular relevance for calibration process. Techniques that make use of repeat pass images to monitor the overall gain variation as well as calibration of SAR images using natural targets have been extensively outlined in [1], [2], [3].

AAP estimation has been traditionally performed using transponders that are high precision and geolocated devices able to provide, when illuminated by a SAR antenna, a fixed Radar Cross Section (RCS) with an accuracy of about 0.1dB and stability in amplification of less than 0.1dB, [4]. The transponders are able to provide what can be considered as close as possible to the impulse response of a white target, so AAP can be easily estimated from them. However the use of transponders is difficult and expensive since they require maintenance, accurate and complex calibration to confirm their RCS and are able to provide just the 1-way gain of the AAP. Moreover they can be located only where backscattering allows a sufficient contrast (i.e. dark background), denying this way the continuous monitoring of the antenna and interfering with the mission requirements.

The natural targets we use in this paper have instead the following features:
1) They are sparse nearly over all the acquired images, allowing a possible extraction of the AAP from nearly all the acquired images;
2) Dense, i.e. many targets, especially in acquisition over cities or man-made targets, can be present in a single image, making the estimation robust;
3) Stable in time; this allows also to estimate the spectral signature of the target and to remove it from the spectral model of the response, even when the target is not white.

A statistical formulation of the problem is here presented since point target amplitudes, background clutter and thermal noise can be well characterized in a stochastic framework. The problem is solved by means of the Maximum Likelihood estimator in which the shape of the azimuth antenna, spectral shape of the target and antenna pointing (the so-called Doppler centroid) are determined using repeated pass images on the same area.

1.1. The target model in a multi-image context

Synthetic Aperture Radar acquisition and focusing can be modeled as a complex source (the ground reflectivity) passed through a linear time-variant (LTV) system (the SAR impulse response) which smears out the energy of a single point scatterer. The inverse processing, said focusing, tries to focus the point scatterer response back to a single point.

Even if focusing is a 2D correlation of acquired data with the SAR impulse response, it can be approximated as two consecutive passages by adaptive filters: the range compression and the azimuth compression.

Let's suppose that 2D compression has been successfully applied to a SAR image; for a single target the spectrum can be written as

\[ Z(f) = S_{sc}(f)A_{ant}(f) \cdot \exp[j\phi(f)] + W(f) \]  

where we can identify

- \( A_{ant}(f) \), the azimuth antenna pattern. It is well known that the spectrum of a ideal pointiform target after compression is just the AAP. AAP is the supposedly unknown, even if we have an approximate model of it.
- \( \phi(f) \) the target phase. This phase accounts for both the uncertainties in the exact estimation of the Doppler centroid (i.e. the mean of the Doppler frequency), \( \phi_f \) and the intrinsic target’s phase. Residual errors, that can be statistically characterized, shall be due to residual in the correction of the optical path from the target to the sensor: uncertainties in the knowledge of the target height over a reference DEM (or ellipsoid), deformation at the acquisition time, atmospheric phase screen (APS).
- \( S_{sc}(f) \), the spectral shape of the target, in the frequency domain. If the target is ideal, its spectral shape is the constant 1, provided that the SAR image has been coarsely normalized.
- \( W(f) \) is a zero-mean complex circular distributed Gaussian noise due to thermal, clutter, processing errors, etc.

The previous model is complicated by the fact that SAR is pulsed in azimuth direction; this makes the spectrum in (1) folded. Part of the clutter noise in \( W(f) \) is composed hence by the spectral replica of other targets that superimpose to the one of interest.

In nominal conditions the antenna shape is difficult to unveil and accurate pointing estimation difficult to obtain. Therefore the method is in practice applicable only when aliasing due to azimuth sampled acquisition is not drown into the distributed backscattering and so just for isolated targets with a strong response, so that the side lobes can be solved within the background.

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For SAR data the effective azimuth bandwidth, which can be extended also over the main lobe, is greater than the actual sampling frequency. Since we want a proper representation of the antenna (i.e. without ambiguity), the sampling frequency must be increased.

Azimuth oversampling can be performed before azimuth focusing with usual oversampling methods but accounting for the time-variant nature of the phase history. With the upsampling the complete history of the point scatterer is reconstructed after focusing, even if other spectral replica are as well generated. This process is called digital spotlight focusing [5] since it applies spotlight method to stripmap data and is used to determine the response with higher resolution for strong and isolated targets.

Now we suppose to have available a stack of $N$ images acquired by repeated geometry, properly co-registered w.r.t. a master reference and where a set of $P$ targets have been properly selected on the basis of their stability (i.e. they are present in all the images of the stack) and whiteness, i.e. their spectrum is very similar to the ideal antenna shape, so they result very “point-shape”,

$$Z_{n,p}(f) = S_{sc,n,p}(f)A_{ant}(f) \cdot \exp[j\phi_{n,p}(f)] + W_{n,p}(f)$$  \hspace{1cm} (2)

These targets have been spotlight-focused, so that their spectral behavior includes a not aliased version of the Azimuth Antenna, at least for a number of sidelobes that usually is 3 or 5, having focused at frequency which is $3X$ or $5X$ of the stripmap Pulse Repetition Frequency (PRF), respectively.

In Fig. 1 is presented an example of the azimuth spectrum of a spotlight-focused ($X5$) point target, to be compared with its Stripmap (with nominal PRF) focusing.

![Figure 1: PSD of a point target retrieved from a CSK image after Stripmap focusing (in red) and spotlight-focusing at 5X PRF (blue). Spectrum in red is to be repeated with PRF period.](image)

1.2. The problem statement

In the previous multi-image, multi-target model a set of unknowns are identified. In the following we discuss the importance of estimate each parameter. The estimation is based on a statistical approach, the Maximum Likelihood estimation. In Fig. 2 is sketched a block scheme of the estimation steps performed in the paper.
1.2.1 Accurate Doppler Centroid estimation

In determining the model in (2) we have supposed that azimuth focusing operator is known with infinite accuracy and that it matches exactly at target distance, while a sensor attitude uncertainty is still present. The accurate estimation of this parameter, i.e. the Doppler centroid, is of fundamental importance for final image resolution and signal-to-noise ratio. Estimation of \( f_{dc} \) from data is not new in literature and has been solved some decades ago [6], [7]; it is however always up-to-date, because of the need of achieving accurate location of targets and a better SNR, especially for a new generation of large bandwidth sensors.

We propose a Maximum Likelihood estimator in which the residual of Doppler centroid, \( \delta f \), is the optimal spectral shift of the target Power Spectral Density (PSD). All the almost-point targets in the image are exploited, as we suppose that the residual centroid is the same for them. The estimate shows to be more accurate than the traditional correlation-based methods and near to the theoretical Cramer-Rao Bound.

1.2.2 Target spectral shape estimation

Estimation of Azimuth Antenna Pattern cannot be simply solved if we consider that real targets own a spectral shape of unknown entity, \( S_{sc}(f) \). We must then estimate the target spectral shape and remove it before to estimate the AAP. Estimation grounds on the fact that the same target, acquired
for a stack of images, owns the same spectral shape on all the images while the clutter noise, which is independent on the same target for different images, is averaged out. LMS polynomial approximation of the spectral shape is possible after that an AAP shape is removed from the model in (2).

1.2.3 Azimuth Antenna Pattern estimation
After correction of the PSD residual shift due to $\Delta f$ and after the removing of the target spectral shape, a multi-image AAP estimation can be carried out frequency-by-frequency. The Likelihood can be written, for each frequency interval, for the set of data referencing to the same target in the image stack; the total Log Likelihood shall be the composition of the Log Likelihoods of the targets stack.

The optimal value of the Azimuth Antenna Pattern, for each frequency bin, shall be the value that maximizes the joint Likelihood for all the targets and for all the images, once that compensation for the previous parameters estimates has been applied.

2. ML Estimation of parameters
2.1. Doppler Centroid estimation
The discrete formulation of the model stated in (2) for a given image (index $n$ is fixed) is achieved by sampling the azimuth spectrum:

$$Z_{n,p}(f_m) = S_{sc,n,p}(f_m)A_{ant}(f_m) \cdot \exp\left[j\phi_{n,p}(f_m)\right] + W_{n,p}(f_m) \tag{3}$$

In the previous we supposed that

$$A_{ant}(f_m) = A_{id}(f - \Delta f) \tag{4}$$

i.e. the antenna in (3) is the ideal version of the azimuth antenna, shifted of the residual $\Delta f$, since the antenna centering w.r.t. each individual target $f_{dc,n,p}$ is supposed to be already performed. $\phi_{n,p}$ contains the unknown parameter to estimate, $\Delta f$. $W_{n,p}$ is the noise (clutter) on the different targets; it can be considered i.i.d. with null mean and std that can be estimated from small window taken around the target itself.

In (3) unfortunately the target spectral shape, $S_{sc,n,p}$ is actually a nuisance parameter and we’d like to avoid it. In a first approximation we can suppose $S_{sc,n,p}$ constant, accounting the not-ideality of each target by weighting the log-likelihood of each target by its quality factor, i.e. the normalized root mean square error of the target spectrum w.r.t. the ideal antenna, summed on all the frequencies

$$q_{n,p} = \frac{\sum_m \left| \hat{S}_{z,n}(f_m) - A_{id}(f_m) \right|^2}{\max_n \sum_m \left| \hat{S}_{z,n}(f_m) - A_{id}(f_m) \right|^2} \tag{5}$$

The quality factor can be considered a sort of Maximum Likelihood Ratio Test (MLRT) [10] for the PPS, and is needed to manage the meaningfulness of the concept of white target.

For each image $n$, SAR data observations $Z_{p,m}$, are usually considered samples from a multivariate Gaussian; so their joint probability, conditioned to $\Delta f$, has a closed-form expression:
\[ p(Z_1,...,Z_P | \tilde{\theta}) \propto \exp\left(-0.5Z^H C_z^{-1}Z\right) \frac{1}{|C_z|} \]  

(6)

Where \( C_z = E[Z^H Z] \) is the covariance matrix and \( |C_z| \) its determinant. The parameter estimation is carried out by maximizing the log-likelihood (LLH). Due to the statistical nature of the data, it can be shown that the covariance matrix is almost diagonal and its elements are basically the power spectrum density of the target \( C_z = \text{diag}[S_z(f'; \tilde{\theta})] \). This lead to a very simple formulation of the LLH for each target:

\[ \ell_{n,p}(\tilde{\theta}) \approx -\sum_m \frac{|Z_m|^2}{S_z(f_m; \tilde{\theta})} \]  

(7)

which can be interpreted as the power of the target whitened spectrum.

The CRB can be found by numerical computation, since for jointly circular Gaussian process its assumes a simple form (see also [7]):

\[ \sigma^2_{\text{CRB}} = -\text{tr}\left( \frac{\partial C_z}{\partial \tilde{\theta}} C_z^{-1} \frac{\partial C_z}{\partial \tilde{\theta}} \right) = -\text{tr}\left( \sum_m \left( \frac{S_z'(f_m)}{S_z(f_m)} \right)^2 \right)^{-1} \]  

(8)

where \( \text{tr} \) is the trace of the matrix. (8) can be numerically solved by means of the well known sinc⁴ shape for the antenna PSD with a number of lobes suitable with the spotlight resolution adopted (usually 3 or 5X the Stripmap sampling frequency); the result is also dependent on the background clutter level w.r.t. the target power. In (8) it appears that the most relevant spectral contributions to the estimation are the ones with high derivative and low power, i.e. the spectral parts close to the nulls. The same equation, applied for the aliased version of the antenna shape in the usual PRF sampling, allows to get the CRB for the Stripmap case. What we found is that, as expected, the limit for \( \text{SNR} \to \infty \) is

\[ \sigma^2_{\text{CRB}} \approx 0.2516 \frac{\text{PRF}}{N_{\text{samp}}} \]  

(9)

already achieved in [7].

2.2. Target Spectral Shape estimation

Let’s suppose that the estimation of the residual Doppler centroid has been performed as in Par. 2.1 and the PSD residual shift has been corrected for each image of the stack. We now have, for the target spectrum, still the model of (3) but the antenna is correctly centered around the zero frequency. To perform a first rough estimation of the target spectral shape it is important to align the data concerning a single target through the stack of images: data are aligned in time and phase. This is done for each of them by estimating a linear second order phase of the spectrum, which corresponds to a fine azimuth peaks registration and a possible residual focusing operator error, and a residual phase offset. After this step we can suppose that the phase term includes just the residual errors due to DEM deformations, APS ad processing. This bring to a new formulation of (3), where the phase contribute in the frequency domain, \( \theta_{n,p} \), becomes hence marginal:
Azimuth spectrum is then the product of antenna and target spectral shape; unfortunately we do not have information on the single contribute of each of them, but we have many measures of the same target on many images that allow to average out the effect of the noise. The determination of the spectral shape of each target can be performed by a LMS polynomial approximation of the residual in the frequency domain, once that a model for the antenna is available.

Lacking any kind of information, we use as model for the antenna the average of all the targets in all the images, each target weighted by its quality factor. We have then:

\[ A_{id}(f_m) = \frac{\sum_n \sum_p q_{n,p} Z_{n,p}(f_m)}{\sum_n \sum_p q_{n,p}} \]  

(11)

which is the ideal (even if raw) antenna model. Now let’s take

\[ Y(f_m) = \frac{\sum_n q_{n,p} \cdot Z_{n,p}(f_m)}{A_{id}(f_m)} \]  

(12)

avoiding just the frequencies too near to the antenna nulls (where noise is the predominant effect). What we get is the model of the target spectral shape, that can be approximated by a LMS polynomial of a given order (usually 2nd). The longer is the temporal stack of the images, the more accurate is the estimation of the target spectral shape.

Please note that this can be considered the first step of a sort of Expectation-Maximization algorithm (see also Fig. 2) [10], since once that a more accurate model of the antenna is available, it is possible to use this in Eq. (12), refining the model of the latent variables (here represented by the target spectral shape at various frequencies) converging so for the unknown parameters (the AAP).

2.3. Azimuth Antenna Pattern estimation

Estimation of Azimuth Antenna is performed frequency by frequency, once that we have available, for each target, an estimate of the spectral shape. Taking again (10), for each frequency \( f_m \) we can write the join PDF

\[ p(Z_1, \ldots, Z_p | A_{ant}(f_m)) \propto \exp\left[-0.5Z^H C_z^{-1}Z\right] \]  

(13)

conditioned to the best value of the antenna model at that frequency. This value is estimated by maximizing the log-likelihood,

\[ \hat{A}_{ant}(f_m) = \arg \max_{A_{ant}} \{ \ell(A_{ant}) \} = \arg \max_{A_{ant}} \left\{ \sum_n \left[-Z^H C_z^{-1}Z - \log C_z\right]\right\} \]  

(14)

The extension to multi-channel (i.e. multi-image) is not straightforward and we simplified the approach by approximating the log-likelihood of \( N \) output as the sum of the log-likelihoods. This approach for a reasonable number of targets and images gives the same level of output accuracy of the
strict ML formulation. Solution can be found by exhaustive search, testing relatively little deflections from the ideal (expected) value of AAP.

The main problem of applying (14) is to have a reliable estimation of $C_e$. It is for this reason that simplification of (14) are used, not based on the inversion of the autocovariance matrix. Impact on the final estimate of the possible simplification are still under study and some preliminary result is shown in par. 3.3.

3. Results

3.1. Doppler Centroid estimate results

Results have been achieved both on a set of simulated point targets in a noisy context and for real X-band SAR data (a CSK dataset).

Targets have been simulated using system and geometrical parameters of Cosmo SkyMed (CSK) satellites constellation, [9]. In CSK yaw steering is adopted in acquisitions, so a relatively low $f_{dc}$ is expected and has been simulated too (i.e. within [-PRF/2,PRF/2] interval). Since in a simulation context the $f_{dc}$ is perfectly known, a wrong value has been intentionally applied during spotlight focusing to simulate an incorrect $f_{dc}$ knowledge after geometric (or traditional) estimation. The estimation of residual $f_{dc}$ has been performed by means of the maximization of (7), by varying $\delta f$ in a suitable interval of research.

The main problem of applying (7) has been to have available an estimation of $S_c(f)$. For high SNR (low clutter on the targets) the covariance matrix is expected to be very good and we use the average of the spectra to estimate it; in tests conditions with low SNR we take instead the ideal model, i.e. the target ideal PSD, as the measures became unreliable. We reported in Fig. 3 the LLH of a set of simulated targets for a SNR of 20 and 30dB. The joint LLH has been achieved by summing the LLH and weighting each of them by their quality factor as defined in (5):

$$\ell_{tot,n}(\delta f) = \sum_p d_p \cdot \ell_{n,p}(\delta f)$$

(15)

Repeating simulations allowed to get also the standard deviation of the estimate and compare it with the theoretical bound provided by the CRB of (8). In Fig. 4a the standard deviation of the estimate vs. the SNR, compared to the two possible traditional correlation-based algorithm.
Valuable results have been achieved on CSK data too. Targets have been chosen as those with a quality factor over a given threshold, so that for a high threshold we assure to handle targets with a PSD shape very similar to the ideal antenna model. Between over than one thousand of possible candidates, 12 targets with an average SNR of 20.8dB were taken. After spotlight processing, the joint LLH of the set was obtained by a weighted sum of the single LLH, as in (15).

In both simulated and real data the 2nd order polynomial approximation is a valid aid to clearly address an accurate and noiseless position of the parameter to estimate. We repeated the experiment using just the 6 best targets and the result did not change sensibly, confirming that the method is very robust to the use of few targets, potentially less than 10 if they own a good SNR, let's say over 20dB.

3.2. Preliminary results on estimate of the spectral shape

Results have been achieved on a set of simulated point targets in a noisy context. Simulation condition and parameters have been already described in par. 3.1; in this experiment the targets have been even distorted in their spectra so to have a non-white spectral shape to estimate.

Since images have been rawly normalized we expect to express the target spectral shape as a deviation from the nominal value of 1 (or 0dB). In Fig. 5b is sketched the result after a simulation of 16 targets whose spectral shape have been pre-distorted. As can be seen the distortions applied to the target spectral shapes sweeps over 1dB; final error is well contained, for the central part of the spectrum, within the ±0.2dB, which can be considered reasonable, noticing that the ideal antenna model is not yet known and that a successive iteration could further improve the results.

In Table 1 is finally reported the mean and the root mean square error (RMSE) of the error in estimating the spectral shape of the target. Error, reported in dB is comparable with the level of added clutter on the simulation, as we have simulated targets with a SNR of 35dB.
3.3. Preliminary results of AAP estimation

Just preliminary results of AAP estimation are here shown. We simplified the LLH estimation by exploiting the (10) instead of the (14); this allowed us to avoid the estimate of the autocovariance matrix of the observations, which is at the present state of the work, an unsolved problem. The problem has been divided in a subset of frequencies: for each subset we coherently summed (10) after correction for the estimated spectral shape, and looked for the most probable value of $A_{ant}(f_m)$ by exhaustive way, as suggested in [8]. In Fig. 6 a first result for the antenna pattern, compared with the first model adopted for the spectral shape estimation and with the ideal model are shown.

![Graph](image)

Figure 5: Target Spectral Shape Estimation.
(a) Simulated spectral shape for 6 targets; (b) estimation error across azimuth frequencies.

Table 1. Mean error and root mean square error in the target estimation for 16 simulated targets [dB]

<table>
<thead>
<tr>
<th>Target #</th>
<th>Mean Error [dB]</th>
<th>RMSE [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.065235</td>
<td>-30.739037</td>
</tr>
<tr>
<td>2</td>
<td>-0.072028</td>
<td>-30.731966</td>
</tr>
<tr>
<td>3</td>
<td>0.001856</td>
<td>-34.440273</td>
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<tr>
<td>4</td>
<td>-0.050133</td>
<td>-31.893612</td>
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<tr>
<td>5</td>
<td>-0.056147</td>
<td>-31.019724</td>
</tr>
<tr>
<td>6</td>
<td>-0.004094</td>
<td>-34.618851</td>
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<td>-0.056134</td>
<td>-31.292093</td>
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<td>8</td>
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<td>-35.888153</td>
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</table>
4. Conclusions

In this paper a method to estimate both the Azimuth Antenna Pattern and its pointing, i.e. the Doppler centroid, for stripmap SAR data has been presented. The method is based on a joint LLH estimation and spotlight processing of natural high-SNR point targets. The algorithm shows to be robust; in particular preliminary results on real and simulated data for antenna pointing show that the algorithm is better than any other correlation-based traditional estimator.

At the present state of the work the method still suffers of target spectral shape distortion, which are nuisance parameters, but that need to be estimated for a correct antenna model estimate; for this reason a LLH estimation of the AAP is still under study and is part of future works.

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![Graph](image)

**Figure 6**: Azimuth Antenna Pattern Estimation in the 5X PRF domain.

(a) (hardly distinguishable): Ideal model (black curve); model used to estimate the target spectral shape, as in (11) (red), estimation as described in par. 3.3 (blue), after a single iterate. (b) Detail around zero frequency.

References


