Geoinformation for European-wide Integration, Benes (ed.)
 ISBN 90-77017-71-2

Dynamic scaling in the environment and remote sensing observation

L.N. Vasiliev
Institute of Geography Russian Academy of Sciences, Moscow, Russia

Keywords: precipitation, density of states, dynamic scaling

ABSTRACT: This paper is an attempt to present a picture of dynamical properties of global precipitation. The implication and applicability of the dynamic scaling hypothesis in precipitation is discussed. The availability of One – degree daily data (1DD) of global precipitation has allowed numerical computations to test the scaling theory for precipitation on a global scale. The spectral dimension is a key dimension for dealing with the dynamical properties of a fractal network, in addition to the fractal dimension. The fractal dimension describes how the mass of the geometrical object depends on its length scale, whereas the spectral dimension characterises the vibrations on a fractal network, such as the density of states (DOS). Some features of the DOS are presented such as the vibrational density of states on a percolating network of precipitation. The concept of scaling, which is generalization of the respective ideas applicable to static properties, provides a framework for understanding a large number of dynamic phenomena.

1 INTRODUCTION

The advances of physics have enriched all branches of science including geophysics and remote sensing and offered a new approach for improving our understanding of the environment. In remote sensing recent theoretical achievements have put forward new questions about understanding complex systems and set the stage for future modelling and predicting non-linear phenomena. Nonequilibrium processes in global precipitation are still not clear enough on scales ranging from the microscopic to the macroscopic level. Regular space measurements on the global scale permitted studies of dynamical processes in the environment. As a result, multiple space-time series are developed. They could be most useful for investigating critical phenomena in the environment. One of the central problems here is associated with the necessity to understand interactions at various scales when local changes may bring about the development of avalanche-like dynamics. Similar phenomena are explained in the model of Self-organized criticality. The ideas of scaling in spatial structures of 3D geophysical fields of different origin have already been frequently confirmed. Such confirmation, however, refers to static / frozen structures. Besides, there is a need to learn how to combine the dynamic effects of processes on different space and time scales in the presence of enormous natural heterogeneity. This is a relatively new and rich area for studying. When a system is at a critical point or close to it, the system suffers anomalies both in dynamic and static properties, which is a subject for thorough research in the theory of critical phenomena. The implication and applicability of the dynamic scaling hypotheses will be discussed in this paper. A hallmark of dynamic scaling approach is the density of states (DOS). A typical feature of the approach suggested is determination of the process characteristics in the frequency domain. The dynamics of a spatial structure appears as vibrations on a fractal network. Such a scenario permits a set of frequencies at each point in space to be characterized by the density of states and its dependence on the spectral dimension d_s. The aim of this research is to assess dynamic scaling of global precipitation from satellite observation within the Global Precipitation Climatology Project (GPCP) Huffman et al (2001). The paper develops further the idea of dynamic scaling, applying it to environmental systems in a far-from-equilibrium state. This subject was already discussed at the EARSeL Symposia. Those interested may address publications Vasiliev (2000, 2002) to get acquainted with the problem statement. We shall discuss a purely phenomenological approach known as dynamic scaling. The example of dynamic scaling is drawn from global precipitation at different length scales.

2 DYNAMIC SCALING

Global precipitation is an example of a non-linear dynamical system. New approaches to this process and 3D space-time pattern formation are associated with a connectivity phase transition in directed percolation Stanley et al (1982). They usher in new possibilities in studying space-time interaction in global precipitation. The modern theory of dynamic critical phenomena provides a way of understanding the detailed behaviour of a vibrating network on an infinite percolating cluster. Here there is a number of characteristics of the system: behaviour in the frequency domain; the vibration density of states, crossover, scaling of a correlation function – all of them are coming together. The theory of dynamic scaling seems rather useful for studying phenomena of different origin, phenomena that reach their critical points via a connectivity phase transition. If we are to get dynamic characteristics of the spatial structure considered, we should determine the dependence of the vibration density of states (DOS) $D(\omega)$ on the frequency ω, which may be written in the form:

$$D(\omega) \propto \omega^{d_s} \quad (1)$$

The exponent d_s, called the spectral dimension, depends on the geometrical properties of a fractal structure. The DOS per one site at the lowest frequency $\Delta\omega$ for this system is written as

$$D(\Delta\omega, L) \propto 1/L^D \, \Delta\omega, \quad (2)$$

where D is a fractal dimension of the structure of a size L. The spectral dimension d_s is a key dimension for describing the dynamical properties of the fractal network in addition to the fractal dimension D. D describes how the mass of the geometrical object depends on its length scale, whereas the spectral dimension characterises vibrations on the fractal network, such as the density of states.

Another useful characteristic associated with the dynamic scaling follows from the correlation function. In terms of scaling it may be written as:

$$C(r,t,t') \equiv \langle \sigma(x,t)\sigma(x+r,t') \rangle = f(r/L', L/L') \quad (3)$$

where $L = L(t)$ and $L' = L(t')$ are linear dimensions of the spatial structure considered at the time t and t' in the state x.

3 DYNAMIC SCALING IN PRECIPITATION

The particular way of dealing with dynamic scaling presented below is intended to illustrate – with a simple example – some of the features of the density of states in precipitation. The starting point is an example of the one-dimensional schematic illustration of precipitation (Fig. 1). This is equivalent in its essence to rain gauge measurements. On the time axis we mark the points corresponding to the start of rainfall, t_s and to its end, t_e. The duration of events is:

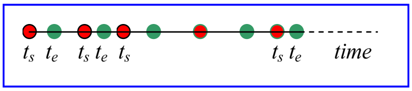

Figure 1. Schematic illustration of precipitation in one-dimension. Starting points of rainfall t_s are shown in red, while ends of rainfall t_e are in green. The frequencies of events are defined by $\omega = \Delta t^{-1}$.

$$\Delta t = \begin{cases} t_s - t_e, \text{for rainfall} \\ t_t - t_s, \text{without rainfall} \end{cases}$$

As a result the time axis will be divided into a set of intervals of different duration, Δt. Events recorded during rain gauge measurements at a meteorological station consist of a sequence of intervals of various lengths, which state the duration of rainfall and periods without precipitation. Thus on the time axis the moments of beginning and end of events may be marked. Here 'event' means the duration Δt of each interval. Thus the frequency ω is given by $\omega = \Delta t^{-1}$ and the DOS of precipitation is described by $D(\omega) \propto \omega^{d_s}$, where d_s is the spectral dimension. The DOS and the spectral dimension were used to characterize such non-linear phenomena as vibration and diffusion on a percolating clusters where the exponent was derived from computer simulation Hohenberg & Halperin (1977), Nakayama, Yakubo and Orbach (1997).

Before we go to global precipitation we consider the effect of applying the idea of the vibrational density of states to our one-dimensional example of rain gauge measurements. Fig. 2 and 3 show examples of employing this approach to determine the DOS of precipitation using rain gauge data in Temperate Zone, in forested and semi-arid areas of Siberian part of Russia over 49 years from 1936 – 1985 with the time interval 12 hours, i.e. the highest frequency is $\omega_H = 1/(12 \text{ hours})$.

The difference in the behaviour of rainfalls in moderate and semi-arid regions may be followed when we study the values of the spectral dimension d_s, which appears sensitive to geographical conditions and to the distribution of precipitation. The log - log plots permit two dependences $D(\omega) \propto \omega^{2.45}$ and $D(\omega) \propto \omega^{1.68}$ to be seen distinctly. At the Alexandrovskoe station ($79.9° \text{E}, 60.4° \text{N}$) (Fig. 2) the DOS is divided in two frequency regions $[1/\Delta t, 1/2\Delta t]$ and $[1/3\Delta t, 1/30\Delta t]$ at the crossover frequency $\omega_c = 1/3\Delta t$. The region in the vicinity of the crossover frequency ω_c, is the crossover from the air

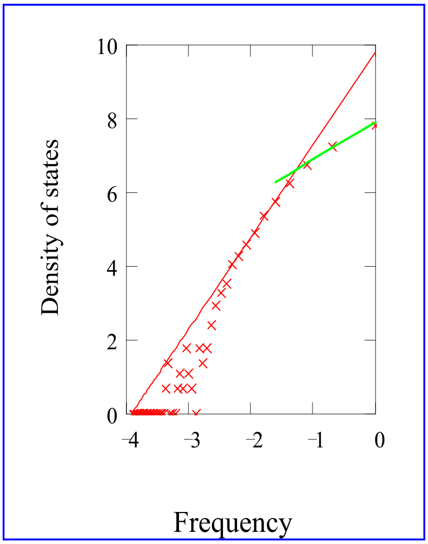

Figure 2. Density of states for precipitation in Alexandrovskoe on a log-log scale. A forest zone in Siberia.

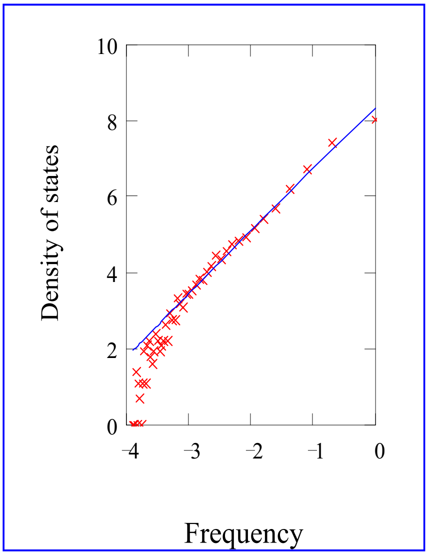

Figure 3. Density of states for precipitation in Irtyshsk on a log-log scale. A semi-arid zone in Siberia.

mass shower regime to rains produced by organized systems. The difference results from the dissimilarity of mechanisms forming short- and long-duration rainfalls, which depend, in turn, on the types of atmosphere circulation: cyclonic and anticyclonic precipitation, mainly of convective type. In the semi-arid zone, however, at the station Irtyshsk ($75.4°$ E, $55.4°$ N) the crossover frequency is not observed.

The idea of vibration on a percolating network may be used to interpret a space-time series of global precipitation. They are presented in an amount of daily precipitation at one-degree daily resolution images (1DD) developed within the Global Precipitation Climatology Project (GPCP) Huffman et. al. (2001). We consider a precipitation process in the space-time coordinate system X, Y, t. If precipitation is recorded in the time interval Δt with the frequency $\omega = \Delta t^{-1}$ as the images described above, they can be defined as a 3D array of precipitation $P(X,Y,t)$. A sample for a fixed point X, Y is equivalent to a rain gauge measurement. Naturally, patterns formed by precipitation at a fixed occurrence rate over the Earth's surface are strongly dependent on their location. As has already been proved Vasiliev (2002), global precipitation forms in space-time an infinite percolation cluster. Therefore for each point of the Earth's surface the DOS may be determined, similar to those shown in Figs. 2 and 3. For that purpose 'start' and 'end' points of precipitation t_s, t_e, t_s, t_e, …are marked in the 3D array $P(X,Y,t)$ along the directions parallel to the time axis t. They will form a pattern with a distinctive distribution of events as described above. A 3D configuration is shown in Fig. 4. A set of elements $\Delta t = t_s - t_e$ presents all events in the frequency domain ω. Cells corresponding to 'start' and 'end' of events occupy 0.3 of the entire volume. Since this value exceeds the critical number $p_c = 0.282$, the occupied cells form an infinite percolation cluster. Only a small group of events will form separate isolated clusters.

The results of determining the DOS for global precipitation from one-daily-degree data over 1997 are shown in Fig. 5. The value of the spectral dimension can be deduced from the slope of the observed DOS versus frequency in the double-logarithmic plot. The frequency dependence of the DOS is split up into two regions at the crossover $\omega_c = 1/(6\,\text{days})$

$$D(\omega) = \begin{cases} \omega^{1.65}, \omega < \omega_c \\ \omega^{2.98}, \omega > \omega_c \end{cases}$$

For Torrid Zone the DOS of precipitation demonstrates considerably different spectral dimensions (Fig. 6)

$$D(\omega) = \begin{cases} \omega^{1.80}, \omega < \omega_c \\ \omega^{2.63}, \omega > \omega_c \end{cases}$$

at the crossover $\omega_c = 1/(9\,\text{days})$.

The above-derived DOS estimates characterize the behaviour of global precipitation relying upon the one - degree daily data. To better understand scaling in the DOS we compare the estimates derived from space borne measurements when the

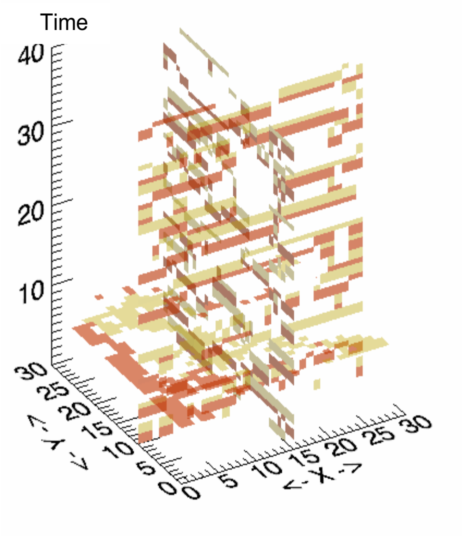

Figure 4. Similar to the one-dimensional case there is a 3D pattern of events in the space-time global precipitation derived from 1DD data for the three years 1997-1999. The picture is a subset showing only the region which dimensions are $30° \times 30° \times 40$ days, centred on $25°$ E, $35°$ N .

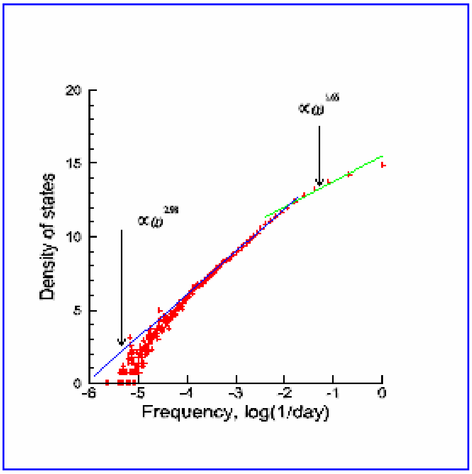

Figure 5. Log-log plot of the density of states for one-degree daily precipitation in 1997. The results show that the frequency dependence of the DOS is characterized by two regimes of precipitation. The region in the vicinity of crossover frequency ω_c is the crossover between air mass shower regime and rains produced by organized systems.

amount of precipitation is taken over the area 10^4 km^2, with the rain-gauge measurements. We address to the rain-gauge data taken over the Siberian region of Russia (see Figs. 2 and 3). The results are summarized as

Space borne data

$$D(\omega) = \begin{cases} \omega^{1.78}, \omega < \omega_c \\ \omega^{2.71}, \omega > \omega_c \end{cases}$$

$$D(\omega) = \omega^{1.78}$$

Rain-gauge data

$$D(\omega) = \begin{cases} \omega^{1.68}, \omega < \omega_c \\ \omega^{2.45}, \omega > \omega_c \end{cases}$$

$$D(\omega) = \omega^{1.68}$$

Low resolution of precipitation measurements leads to an increase in the spectral dimension d_s. The region in the vicinity of ω_c is a crossover region between the air mass shower regime and rainfalls produced by organised systems such as fronts and low centres. It should be emphasized that the DOS are smoothly connected in these regions, exhibiting no notable steepness. The spectral dimensionality increase depends on the frequency region of the DOS. Hence the crossover ω_c can be thought of as dimensionality change corresponding to the transition from the air mass shower regime to rainfalls produced by organised systems. However, the theoretical understanding of scaling in the DOS of precipitation is not satisfactory at the present stage. We now try and explain qualitatively the latter relying upon the scaling of the correlation function (4). Fig. 7 demonstrates the behaviour of the correlation function $C(t, L)$ due to transformation of precipitation data from high to low resolution $L \times L = 1° \times 1° \Rightarrow L \times L = 19° \times 19°$. The experimental data of $C(t, L)$ is fitted to a scaling equation:

$$C(t, L) = \exp\left[-\left(\frac{t}{aL}\right)^{\beta}\right], \quad (4)$$

with the exponent $0 < \beta < 1$ and $a = 0.161$. Hence β is characterizing how the correlation function

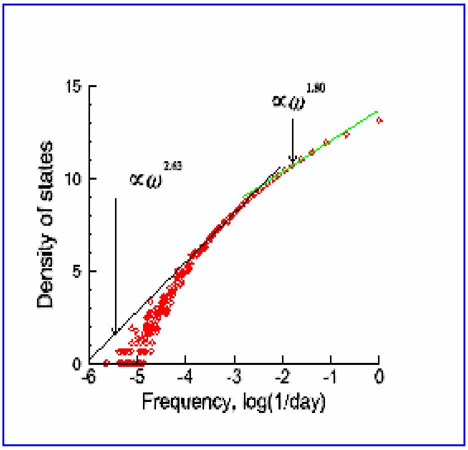

Figure 6. Log-log plot of the density of states for one-degree daily precipitation on Torrid Zone. The differences between plots in the Fig.5 and Fig.6 indicate the relative predominance of convective rain on the Torrid Zone in terms of frequency and show how the crossover shifts.

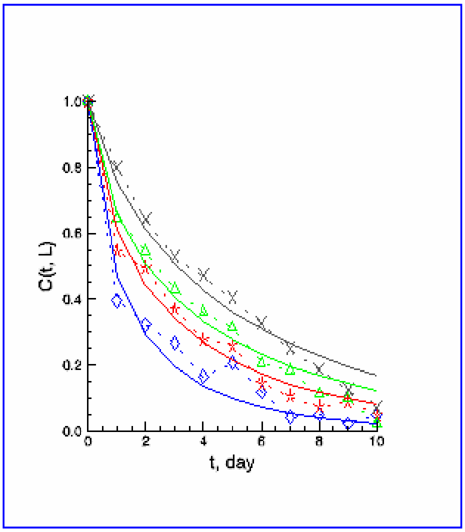

Figure 7. Scaling in the correlation function of global precipitation for four L-lengths. The site on the Equator at longitude 30° west. From bottom to top: $L\times L = 1°\times 1°$ (diamond, blue), $L\times L = 5°\times 5°$ (asterisk, red), $L\times L = 9°\times 9°$ (triangle, green), $L\times L = 19°\times 19°$ (cross, dark grey). $\beta = 0.70$, $\alpha = 0.161$.

scales with increasing "rain-gauge measurement area" $L \Rightarrow bL$. Then we have $C(t,bL) > C(t,L)$. Scaling of the correlation function brings about the transformation of frequencies and, as a result, of the spectral dimension, d_s, as well as the DOS of precipitation. Thus, the increasing in L leads to a d_s-increase, as is obvious from the comparison of the DOS values measured at different spatial resolution. The correlation function should have the scaling form depending on the single length scale L (3). Physically, it is the consequence of interaction between nearest-neighbour cells. The tendency implies that with increasing L the frequency bandwidth decreases. When a certain critical L- length is reached the precipitation within all cells would be recorded every day and the DOS become a meaningless characteristic.

4 CONCLUSIONS

The paper is an attempt to present a picture of dynamical properties of global precipitation. The availability of 1DD data has allowed numerical computations to shed light on and to test the scaling theory for global precipitation. The density of states gives rich information on the dynamics of fractal structure of precipitation. Some features of the DOS were described as a vibrational density of states on a percolating network of precipitation. The concept of dynamic scaling, which is generalization of the respective ideas applicable to static properties, provides a framework for understanding a large number of dynamic phenomena. The density of states may be applied to dynamic models, where it provides a mathematical mechanism for scaling in complete analogy to the static situation. It is obvious that we are now just at the very beginning. The relationship between local description of precipitation and the global scale has not yet been established in full detail. It is not yet clear enough how the global precipitation driven by an external force of frequency Ω reacts.

ACKNOWLEDGEMENTS

The 1DD data were provided by the NASA/Goddard Space Flight Center's Laboratory for Atmospheres, which develops and computes the 1DD as a contribution to the GEWEX Global Precipitation Climatology Project.

REFERENCES

Hohenberg, P.C., Halperin, B.I. 1977. Theory of dynamic critical phenomena. *Reviews of Modern Physics* Vol. 49 (No 3): 435-475.

Huffman, G.J., Adler, R.F. et al. 2001. Global Precipitation at one-degree daily resolution from multi-satellite observations. *Journal of Hydrometeorology,* 2, 36-50.

Nakayama, T., Yakubo, K., Orbach, R.L. 1994. Dynamical properties of fractal networks: Scaling, numerical simulations, and physical realizations. *Reviews of Modern Physics* Vol. 66 (No 2): 381-443.

Stanley, H.E., Reynolds, P., Redner, S. and Family, F. 1982. Position-space renormalization group for models of linear polymers, branched polymers, and gels. In T. W. Burkhardt & J. M. J. van Leeuwen (eds.). *Real-Space Renormalization*. Springer-Verlag, Berlin, 169-206.

Vasiliev, L. 2000. Self-organized criticality in the environment and remotely sensed data. In J. L. Casanova (ed.). *Remote Sensing in the 21st Century: Economic and Environmental Applications*. Balkema, Rotterdam, 255-259.

Vasiliev, L. 2002. Spatio-temporal pattern formation in rainfall from remote sensing observations. In G. Begni (ed.). *Observing our environment from space*. Balkema, Rotterdam, 159-165.