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Development and validation of a sea surface fractal model: A project overview

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ABSTRACT: This paper presents the scientific results of a research project funded by the Italian Space Agency (ASI) and selected, with other seven projects, by the European Space Agency (ESA) to test and validate the first available Envisat ASAR product data (ASAR Wave Mode), in the framework of the "Calibration/validation program" of the Envisat mission. The research makes use of a dynamic fractal model of the sea surface based on the Weierstrass functions (Berizzi *et al.* 1999, Berizzi & Dalle Mese 1999) in order to generate different sea situations from very calm to very rippled sea and it aims to validate this sea fractal model through the analysis and processing of synthetic surfaces, real ERS SAR images, and simulated ASAR Envisat data. SAR image fractal models for oceanographic applications based on Fractal Interpolation Functions (FIF's) and fractional Brownian motions (fBm's) (Pesquet-Popescu & Lévy-Véhel 2002) have be also considered for testing performances of the proposed fractal analysis algorithms.

1 INTRODUCTION

Sea surfaces and electromagnetic scattering from sea surfaces have fractal nature. This is the main result obtained by the scientific community and widely confirmed by the large amount of experimental and theoretical results presented in the literature.

In the next future, thanks to the huge amount of synthetic aperture radar data provided by the Envisat mission, the oceanography and remote-sensing research community will have the opportunity to validate the fractal characteristics of SAR signals and images of the sea surface. The validation procedure may be performed by comparing the results obtained by different fractal and multifractal analysis algorithms applied both to real ASAR images of a sea surface and ASAR-like images produced by a SAR simulator.

In the research project, we generate a sea fractal surface by using the parameters estimated from real locally-acquired data, i.e., in situ data measurements collected by directional wavemeter buoys of the Italian Wavemeter Network (RON); we generate simulated ASAR data in the same geometry conditions of the real data acquisition; we perform fractal analysis of both simulated and real data; finally, we verify the congruence of the estimation results.

Three different algorithms for fractal analysis have been developed and tested: a morphological covering method, a modified box-counting algorithm, and a wavelet-based fractal-dimension estimation algorithm (Berizzi *et al.* 2001). The morphological covering algorithm is able to select automatically the minimum scale and the scale interval under which the morphological volume and the fractal dimension are estimated. The fractal analysis algorithm, based on a modified box-counting method, considers random/arbitrary displacements, in both the vertical and horizontal directions, of the boxes, thus reducing any possible bias effect, especially for low values of fractal dimension. The wavelet technique for 2D fractal dimension estimation shows the capability to select automatically the subbands which have to be considered to compute the linear regression (in log-log plot) of the subband energies of the fractal image. This is made possible by considering a model-based constraint on power ratios, or differences in log scale, between adjacent subbands. A reliable estimator of the local fractal dimension is also described, which allows to obtain a pixel-wise estimation by using a non-decimated wavelet represent-ation.

The experimental results are presented in this paper in terms of estimated fractal dimension, globally or locally computed, fundamental wavelength, and wave propagation direction on couples of SAR images: a real image and a corresponding simulated ERS SAR image.

2 PROJECT ACTIVITIES

In this section we resume the project activities for the first two years of this research.

The first year of the project has been devoted to the following activities:

1) Refinement of the sea surface fractal model and physical interpretation of its parameters.
2) Development of an algorithm for the estimation of the sea model parameters starting from the sea surface spectra measured by the buoys of the Italian Wavemeter Network (RON) or by using the common consolidated models of Pierson-Moskowitz (PM) and Jonswap (JSW).
3) Development of a SAR simulator.
4) Definition of the algorithms for fractal analysis of ASAR-like signals and images.

2.1 *Fractal model*

The sea surface is fractal under certain assumptions for which it can be modeled as a finite sum of sinusoidal waves having a mean propagating direction equal to the wind direction. These assumptions, described in (Berizzi & Dalle Mese 1999) are physically coherent and produce a 2D Weierstrass function model for the sea surface, partially validated in (Berizzi *et al.* 2000).

Signals backscattered by natural fractal surfaces are also fractals within a certain range of scales: this effect has been theoretically demonstrated for 1D electromagnetic (EM) signals scattered from a fractal surface having fractal dimension *D*, with the EM signal maintaining the same fractal dimension *D* of the illuminated surface (Berizzi & Dalle Mese 1999). Something similar seems to happen with 2D data (Berizzi *et al.* 2001), at least on simulated images.

The sea model expression is the following:

$$\xi(x,y,t) = \sigma C \sum_{n=0}^{N_f-1} b^{(s-2)n} \sin\{K_0 b^n [(x+V_x t)\cos\beta_n + (y+V_y t)\sin\beta_n] - \Omega_n t + \alpha_n\} \quad (1)$$

whose model parameters are reported in Table 1.

Table 1. Parameters of the sea surface model.

Time Parameters		Geometry Parameters	
Wave phase	α_n	Fundamental spatial Wavenumber	K_0
Angular frequency	Ω_n	Scaling factor	$1<s<2$ $s = D-1$
Wave prop. direction	β_n	Roughness factor	$b > 1$
Observer velocities	(V_x, V_y)	Number of waves	$N_f >> 1$
		Standard deviation of sea wave height	σ
		Norm. constant	C

The refinement of the sea model consisted in specifying a few technical aspects of the sea model. In detail, we have defined the statistical behavior of the phases α_n and of the propagation angular direction β_n of the sea waves composing the sea and we have evaluated the range of values that the parameters of the model can realistically assume. The phases α_n are considered as independent random variables uniformly distributed in $[-\pi,\pi]$ and independent on the angular directions β_n which have been described by Gaussian random variables with standard deviations decreasing with the sea wavelengths. In other words, larger wavelengths (gravity long waves) have a distribution which is more concentrated around the mean value, i.e. the sea waves mainly propagate along the mean direction. Instead, capillary waves span in all directions and tend to be uniformly distributed.

We have evaluated realistic values of the model parameters by fitting a large number of measured spectra. The results of this analysis are that the roughness factor *s* is always less than 1.33 as claimed in literature and confirmed by experimental analysis, while the scaling factor *b* belong to [1.01, 1.3] and its value regulates the ripple in the equilibrium wavenumber domain. The standard deviation of the sea height is related to the significant wave height h_s by $\sigma=4h_s$. The fundamental sea wavelenght $\Lambda_0 = K_0/2\pi$ depends on the wind intensity. In the Mediterranean sea and for significant wave heights of about 3-5 m, Λ_0 assumes values from 90 m to 150 m.

2.2 *Estimation of model parameters*

From the fractal model surface, new models of the directional sea wave spectrum $W(K,\Phi)$, of the omnidirectional spectrum $S(K)$ and of the spreading function $G(K,\Phi)$ have been obtained (Berizzi & Dalle Mese 1999). Once a reference sea wave spectrum $S_R(K)$ is given (obtained by other models or estimated from data buoys measurements), the sea fractal model parameters have been obtained by minimizing the Mean Square Error between $S(K)$ and $S_R(K)$ under the constraint of preserving the position and the amplitude of the maximum value of the reference spectrum. The following algorithm has been defined:

- Evaluate σ as the root square of the integral of the reference spectrum over K;
- Estimate *s* from the mean slope $\hat{p}$ of the reference spectrum in the equilibrium region by

$$\hat{s} = (5+\hat{p})/2 \quad (2).$$

The value of $\hat{p}$ is computed by linearly fitting the log-log plot of the reference spectrum in the equilibrium range.

- Estimate the parameters ε, K_0, and b by minimizing the mean square error between $S(K)$ and $S_R(K)$ under the constraint of preserving the position and the amplitude of the maximum value of the reference spectrum.

The mean propagation angular directions are estimated by the sea wave direction measurements collected by the buoys or by fitting $G(K,\Phi)$ on a reference one $G_R(K,\Phi)$. The algorithm has been implemented in a FORTRAN numerical code and it will be applied to the Envisat products ASA_WV_2P, when available.

2.3 *SAR simulator*

A sea SAR simulator which uses the omnidirectional sea spectrum of Pierson-Moskowitz and simple spreading functions has been applied. The simulator was formerly developed at the Dept. of Information Engineering of the University of Pisa. The outputs of this simulator are: the sea surface; the reflectivity map; the SAR image in slant-range/azimuth. To make this simulator useful for the objective of the project, the following functionalities have been introduced: 1) The simulator uses the fractal directional sea wave spectrum which is analytically derived from the sea surface model; 2) A data base containing the ASAR parameters has been created; 3) Some new SAR image outputs have been included with SLC images in ground-range/azimuth, PRI images, and multilook images.

Image mode and wave mode ASAR image can be now generated. This allows to simulate ASAR images; real data will be available during the project for experimental results.

2.4 *Algorithms for fractal analysis*

This activity aimed to develop the algorithms for fractal analysis of ASAR-like signals and images. Three different approaches have been considered:
- Box counting;
- Morphological covering;
- Wavelet.

The box counting algorithm basically relies on the following definition of fractal dimension D_B (box-counting dimension) of a fractal surface S:

$$D_B = \lim_{\delta \to 0} \frac{\log N_\delta(S)}{\log(1/\delta)} \tag{3}$$

where $N_\delta(S)$, for a fractal surface S, is the number of boxes of size $\delta \times \delta \times \delta$ required to cover the whole 3-D graph of the $\Re^2 \to \Re$ surface S. D_B , like any other fractal dimension, assumes values between 2 (for flat surfaces) and 3 (for extremely rough surfaces). Obviously, the implementation of the above formula on a computing system requires to be adapted to sampled, discrete functions. The limit is approximated by linear regression on a log-log diagram reporting on its axes the two components of the fraction in (3). For the sake of simplicity, we are unable to provide here an exhaustive description of the solutions used to solve the various problems posed by switching from a continuous to a discrete domain. It is noteworthy that we have modified the box-counting algorithm, by introducing the following functionalities: 1) Good performance can be obtained working on square data matrix with size greater than 256×256. 2) A correction factor which depends on the value of the fundamental frequency is applied in order to compensate for systematic errors in the estimation.

The morphological covering algorithm is based on the Minkowsky-Boulingand fractal dimension (Maragos & Sun 1993), defined as

$$D_{MB} = \lim_{\varepsilon \to 0} \left[3 - \frac{\log(V_B(\varepsilon))}{\log(\varepsilon)} \right] \tag{4}$$

where $V_B(\varepsilon)$ is the morphological covering volume at the scale ε. Given a constant k, eq. (4) can be approximated by

$$\log \frac{V_B(\varepsilon)}{\varepsilon^3} = dim_{MB} \cdot \log\left(\frac{1}{\varepsilon}\right) + k \tag{5}$$

for small values of ε. For a pure fractal, eq. (5) represents a straight line on the plane (x,y), where $x = \log(1/\varepsilon)$ and $y = \log(V_B(\varepsilon)/\varepsilon^3)$. According to (5), the fractal dimension can be estimated by a regression line of the (x,y) plot.

The numerical implementation of the fractal dimension estimator consists in computing the morphological covering volume for different scales ε; plotting all the couple (x,y) of eq.(5); estimating the fractal dimension as the mean slope of the log-log graph thus obtained.

Due to the finite bandwidth and to the discrete nature of the images, a suitable range of scales, under which to evaluate the slope, must be chosen. That range is between the minimum scale (ε_m) and the maximum scale (ε_M).

The minimum scale is chosen as a function of the oversampling factor defined as the ration between the sampling frequency F and two times the image bandwidth B, i.e. $K_s=F/2B$. The sampling frequency F is the maximum between F_x and F_y (sampling frequencies along the x and y coordinates of the image) and B corresponds to the maximum between B_x and B_y (bandwidths along x and y). The scale interval $\Delta\varepsilon$, from which we calculate the maximum scale $\varepsilon_M=\varepsilon_m+\Delta\varepsilon$, depends on the pixel size (Δ_x, Δ_y), on the frequency F_0 of the maximum peak image spectrum and on the image fractal dimension D_I. The dependence on D_I is critical because D_I is the parameter to be estimated. To overcome this problem $\Delta\varepsilon$ is estimated according to the following procedure:

- obtain a rough estimate $\Delta\varepsilon'$ of the scale interval as a function of Δx, Δy and F_0;
- get a rough estimate of the image fractal dimension;
- calculate the final scale interval $\Delta\varepsilon$ as a function of Δx, Δy and the rough estimate of the fractal dimension.

The final fractal dimension estimate $\hat{D}_I$ is obtained from the mean slope of the (x,y) graph in the scale interval $[\varepsilon_m, \varepsilon_M = \varepsilon_m + \Delta\varepsilon]$. The algorithm structure for the 2D case can be easily derived.

The multiresolution decomposition algorithm estimates the fractal dimension of a fractional Brownian motion (fBm) which has been indicated as a proper SAR image fractal model for oceanographic applications (Pesquet-Popescu & Lévy-Véhel 2002). For the 2D case, the fBm is a stochastic process $V_H(x,y)$ with zero-mean Gaussian increments satisfying the following equation

$$E\{|V_H(x,y) - V_H(x+\Delta x, y+\Delta y)|^2\} = \|(\Delta x, \Delta y)\|^{2H} \quad (6)$$

where $0<H<1$ is the Hurst coefficient, or persistence, which is related to the fractal dimension by $D=3-H$. Obviously, a simpler version of (6) holds for the 1D signal $V(x)$ and its 1D increments, but in the 1D case the fractal dimension is related to H by $D=2-H$.

The algorithm is based on the following property of the average power spectrum of a 2D fBm:

$$P(\omega_1, \omega_2) \propto \frac{1}{\left[\sqrt{\omega_1^2 + \omega_2^2}\right]^{2H+2}} \quad (7)$$

Thanks to eq.(7), a dyadic wavelet decomposition of an fBm allows to compute the theoretically constant ratio $R = 2^{2H}$ between the powers of adjacent subbands. From the computation of R, it is possible to determine the value of the persistence H, and, finally, the estimated value of the fractal dimension D. The suggested methodological approach uses a non-decimated dyadic wavelet decomposition, i.e. a space-invariant transformation for which the same sampling rate is adopted at each decomposition level. This approach allows to avoid the problems due to aliasing when non-ideal filters are used in a critically-sampled decomposition scheme.

The estimation algorithm operates as follows. At each level, the image is decomposed into four images: the approximation (LL), and three detail images (HL, LH, HH). The sum of the powers of the detail images is computed, while the approximation is decomposed at the second level. The ratio of powers of the detail images at adjacent decomposition levels are computed, and the fractal dimension D is estimated by inverting $R = 2^{2H}$, as explained before.

Since it has been noted that, in some cases, real SAR images of the sea surface show fractal characteristics only along a particular direction, the wavelet estimation algorithm has been consequently modified. If this condition is verified, the power ratios are computed only from the horizontal, or vertical, or diagonal detail coefficients.

During the project, it has been experimentally shown that the wavelet approach can be adapted to be successfully applied also to 1D and 2D Weierstrass functions.

3 RESULTS AND CONCLUSIONS

3.1 *Simulated fractal signals*

We have generated fractal signals with similar characteristics of ASAR 1D signals having known fractal dimension in order to test the estimation algorithms. A statistical test has been performed by analyzing more than 3000 Weierstrass signals which simulate a great number of real sea surface conditions.

The parameters are described below:

- Fractal dimension varying in the interval [1.1 1.8] with step 0.05;
- 50 realizations for each value of the fractal dimension;
- Fundamental period T_0 = 0.02 sec;
- Number of tones N_f = 50;
- Scale factor b = 1.05.

The results of the statistical analysis include mean (MAE) and standard deviation (STD) of the absolute error, and the probabilities of the relative absolute error to be greater than 5% (Pe(5)) and 10% (Pe(10)). Best results are provided by the wavelet method, for which the estimate is also unbiased. The method is very accurate, as shown in Table 2. For example, MAE = 1.41% when the signal-to-noise ratio is 20 dB.

Table 1. Estimation results of the wavelet method for different values of SNR.

SNR	MAE (%)	STD (%)	Pe(5) (%)	Pe(10) (%)
Inf	0.66	0.84	0.00	0.00
30dB	1.29	1.65	0.53	0.00
20dB	1.41	1.81	0.80	0.00
10dB	3.44	4.37	23.07	2.13
5dB	9.24	11.27	75.20	36.13

3.2 *Real image data*

The validation of the algorithms has been carried out by using the following procedure: apply the fractal analysis tools to the image and perform comparisons of the results; starting from the sea surface parameters estimated by the in situ buoys, generate the simulated ERS1 images, perform fractal analysis and compare the results. The purpose of the experimental phase is to demonstrate that sea SAR ERS1 images possess fractal characteristics, and that there is a correlation between the fractal dimension of the sea, D_S, estimated from *in situ* data, and the fractal dimension of the SAR image D_I, thus allowing an indirect estimate of D_S from D_I.

Theory states that in ideal condition the fractal dimension of the sea and of the images are equal. In real situations, i.e. when shadowing effects, multiple scattering and finite conducibility are present, the fractal dimension of the signal increases. Since the reflectivity map of the surface depends on the fractal characteristic of the electromagnetic scattering, it will have the same fractal characteristic of the signal. Moreover, the SAR image can be considered as a linear filtering of the reflectivity map. Since the impulse response of the SAR is very sharp, the SAR image will have similar fractal characteristic of the reflectivity map and of the scattered signal too.

Three ERS-1 SAR images have been considered together with the corresponding fractal dimensions estimated from buoy measurements at the same dates of the ERS-1 acquisitions. The three images have been acquired around the point 40°52'0"N, 12°57'0"E on 14 Nov. 1992, 3 Apr. 1993, and 26 Dec 1993 at about 9:50 AM. Each image has been cropped to form one 512×512 matrix centered on the buoy. The three sub-images have been named Test1,2,3.

To verify the correctness of the sea fractal dimension D_s, estimated from the buoy data, we have simulated the sea ERS1 image in the same operating conditions of the real ones, starting from the sea surface generated with the estimated fractal dimension D_S. Tables 2, 3, and 4 report the fractal dimension estimates of the sea D_S, of the simulated images D_{SI}, and of the real ERS1 images D_I, obtained by applying the three algorithms described in Section 2.4.

Table 2. Results of BC fractal analysis on SAR images.

	D_S	D_{SI}	D_I
Test1	2.152	2.400	2.660
Test2	2.339	2.430	2.680
Test3	2.286	2.440	2.690

Table 3. Results of MC fractal analysis on SAR images.

	D_S	D_{SI}	D_I
Test1	2.152	2.444	2.393
Test2	2.339	2.486	2.508
Test3	2.286	2.521	2.530

Table 4. Results of W fractal analysis on SAR images.

	D_S	D_{SI}	D_I
Test1	2.152	2.361	2.135
Test2	2.339	2.593	2.339
Test3	2.286	2.475	2.415

From Tables 2-4 we can draw the following conclusions.

- The fractal dimension D_{SI} estimated by the Box counting (BC) algorithm are not reliable because obtained by processing a data matrix whose size 85 x 587 is not sufficient to the box-counting algorithm to have good performance. These results should be ignored.
- The fractal dimensions D_{SI} obtained from algorithms MC (Morphological Covering) and W (Wavelet), are in agreement (show the same trends) with the fractal dimensions D_I of the images.
- The fractal dimensions D_I of the SAR images estimated by the W algorithm are excellent for Test1 and Test2, while the value of Test3 is overestimated.
- The covering algorithm produces an overestimate of the sea SAR image fractal dimension. This effect is due to the presence of noise, which is known (Maragos & Sun 1993) to bias estimates provided by covering algorithms in the sense of increasing them, especially at low fractal dimensions like those usually observed on the sea surface.

To confirm these observations, far more data than the three images should be considered. This is expected to become feasible with the inception of ENVISAT ASAR data dissemination (ENVISAT-AO ESA Exploitation project 161). A large set of ASAR images will be available starting from September 2002 in the framework of the Envisat calibration and validation.

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