New Developments and Challenges in Remote Sensing, Z. Bochenek (ed.)
©2007 Millpress, Rotterdam, ISBN 978-90-5966-053-3

# Comparing distances for quality assessment of fused images

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Keywords: quality evaluation, image fusion, distances, quality budget

ABSTRACT: This communication deals with the fusion of panchromatic (PAN) images of high spatial resolution and multispectral (MS) images of lower resolution in order to synthesize MS images at high resolution. These fused images should be as identical as possible to images that would have been acquired by the corresponding space borne sensor if it were fit with this high resolution. A protocol for the assessment of the quality of the fused images was discussed by the EARSeL Special Interest Group "data fusion" in 2004. It evaluates how much fused images comply with two properties, on multispectral and monospectral viewpoints. The compliance is measured through a set of distances between the set of fused images and the multispectral reference images. This communication analyses the distances that are found in literature. First of all, it proposes a classification of these distances into seven categories. Then it shows some relations between several distances through an empirical study. Finally, a typical choice of distances is proposed in order to assess most aspects of fused images.

## 1 INTRODUCTION

This communication deals with the fusion of panchromatic (PAN) images of high spatial resolution and multispectral (MS) images of lower resolution in order to synthesize MS images at high resolution. These fused images should be as identical as possible to images that would have been acquired by the space borne sensor if it were fit with this high resolution.

As no MS reference is available at this high resolution, quality assessment of fused products is not obvious. Several weak approaches were found in literature which only drew a part of the quality reached by the fusion process, dealing only with some aspects of it. A protocol for the assessment of the quality of the fused images was proposed by Thomas & Wald (2005) to the EARSeL Special Interest Group "data fusion". It combines the two former works of Li (2000) and Wald *et al*. (1997). The particularity of this new protocol is to always refer to MS original images since they are the only genuine references.

This protocol is organized in order to evaluate how much fused images comply with two properties, on multispectral and monospectral viewpoints. The first property consists in measuring the distance between the set of fused images downsampled to the original spatial resolution of multispectral images and the corresponding original set of images; this is the consistency property. The second property, called the synthesis property, deals really with fusion since it concerns the proximity between the fused set and the multispectral reference images at high resolution.

This communication does not raise the important issue of the lack of multispectral images at high resolution to serve as reference since it focuses on the distances for measuring the proximity between references and fused images. Here, the references are the original MS images; the images to fuse are the original PAN and MS images downsampled by a factor of 2 or 4. The fused products are therefore created at the same resolution than the original MS images and they are compared to them. A recent work by Thomas & Wald (2006a) explores this important issue.

Whatever the property to check, the proximity between a reference and a fused product should be measured. In Thomas & Wald (2005), the protocol was completed by a large but non-exhaustive list of distances. The quality is assessed by checking how much some quality criteria are met. Such criteria are function of the distances and are often based on thresholds applying to these distances. For instance, the distance ‘‘correlation coefficient’’ should be greater than the threshold ‘‘0.8’’. Besides this quantitative analysis, a visual analysis should be performed to assess quality.

Distances are called metrics or indices or statistics at times in literature. Many distances can be found in the literature. Each of them has its qualities and drawbacks. Taken individually, distances are not sufficient to describe quality as a whole. This communication aims at exploring to what extent the various distances and their possible combinations are able to provide an effective assessment of the quality. With simple illustrations, we show how some distances can be gathered into categories. Other distances are not independent and produce redundant results; the set of distances can thus be reduced. We use mathematical demonstrations or empirical surveys. Our work deals with global distances that aim at characterizing the overall quality of the fused set. We explored them as well as several combinations of individual distances, forming quality budgets. For example, a quality budget contains only one distance if it is a global one, but may consist of several ones, selected in both monospectral and multispectral categories. This aims at obtaining a more effective indicator of quality. The interpretation of these observations results in suggesting a series of quality budgets.

Section 2 proposes a tour of usual and less usual distances found in literature. A classification into seven categories is proposed. Section 3 uses mathematical demonstrations and illustrations to show dependencies and redundancies between distances. Finally, the conclusion suggests quality budgets, aiming at giving a complete assessment of fused product quality.

## 2 DISTANCES TO MEASURE SIMILARITIES AND THEIR CATEGORIZATION

A distance quantifies the discrepancy between a reference and the fused product. A quality budget is a composition of one or more distances. The distances (respectively quality budget) are of two types: monomodal and multimodal. A monomodal distance (respectively quality budget) applies to a single modality while a multimodal distance (respectively quality budget) applies to several modalities.

Many distances can be found in the literature. Each of them has its qualities and drawbacks. We made an inventory of the distances found in thirty-six articles and communications proposing distances. We counted thirty-nine different distances.

We observe that faced to these huge amount of distances, authors of articles proposing new methods for fusion, evaluate the quality of their products by picking up between 2 and 4 distances among these 39. Beside the visual analysis, the protocol proposed by Wald *et al*. (1997) recommends the analysis of up to 14 distances. We think that this number of distances to evaluate is too large to be commonly adopted by the scientific community as well as providers of fused products. Consequently, we focus on the possible grouping of these distances into a limited number of classes.

The notations are as follows. $B$ denotes the set of multispectral images at original resolution $l$. $k$ denotes the spectral band among the $N$ bands. $B_k$ denotes the image in band $k$. $(i, j)$ are the coordinates of the pixel. In a similar way, $B^*$ denotes the fused product at the same resolution. It is very important that any comparison between a reference and a fused product should be made at the same resolution. It has been proven several times that the content of an image depends on its resolution, including global quantities such as the mean.

This grouping is based on the mathematical analysis of the distances. We found seven categories.

### 2.1 *Category 1 "global measures on images"*

The category 1 is made of distances that evaluate the "difference of global measures on images". As far as monomodal aspects are concerned, these global measures are the means of each image, the variances, standard deviation, entropies of each image. An example is given by the distance "bias", which is the difference between the mean of $B^*_k$ and $B_k$ for the band $k$. Such difference of global measures on images were originally proposed by Munechika *et al*. (1993) and Wald *et al*. (1997) and are very often used. They can also be used as relative values. The difference in means (the bias) is such divided by the mean of $B_k$. Differences in variance, standard-deviation and entropy must be divided by respectively, the variance, standard-deviation and entropy of the original image $B_k$. Another global measure of interest is the mode of the histogram of each image (Ballester *et al*. 2003).

Wald *et al*. (1997) consider the multispectral data set as a whole. They proposed to compute the difference in the number of *N-tuplets* (spectra) found in $B$ and $B^*$. A positive difference means that the synthesized images do not present enough *N-tuplets*; a negative difference means too many spectral innovations.

As these distances deal with differences between fused images and reference images, the ideal value for each of these distances is zero.

### 2.2 *Category 2 "image of difference"*

The category 2 is made of distances that apply to the image of difference between $B^*_k$ and $B_k$ computed at each pixel $(i,j) : B_k^*(i,j) - B_k(i,j)$. The standard deviation and the RMSE are the most known of such distances (Munechika *et al*. 1993). Equivalent are the variance and the mean square error. They can also be used as relative values. The standard deviation and the RMSE are such divided by the mean of $B_k$.

One may consider the absolute value of the error at each pixel instead of the signed error (i.e., positive or negative). Wald *et al*. (1997) propose to compute the histogram of these absolute errors, possibly in relative values, i.e., after dividing by the original value $B_k(i, j)$. It can be seen as the probability density function. Therefore, we can compute the probability of having at a pixel an error lower than a given threshold.

As these distances deal with differences between fused images and reference images, the ideal value for each of these distances is zero. In the case of the probability of having an error lower than a given threshold, this ideal value is 1.

## 2.3 *Category 3 "correlation"*

The category 3 is made of distances that measure the similarity in small size structures between $B^*_k(i, j)$ and $B_k(i, j)$. The correlation coefficient is the most popular of these distances (Wald *et al*. 1997). Wang & Bovik (2002) propose an image quality index, named *Q*. This measure is an attempt to take into account the human perception. It stems from the multiplication of three terms: the correlation coefficient, which is sensitive to the high frequencies of the image, a second term sensitive to the difference in contrast, and finally an element depending on the distortion in grey levels (bias). An improvement was proposed by Piella & Heijmans (2003).

As these distances deal with similarities between fused images and reference images, the ideal value for each of these distances is 1.

Lillo-Saavedra *et al*. (2005) propose an index called the spatial *ERGAS*, in a tentative to mimic the *ERGAS* which is a well-known quantity for depicting the global quality of a fused data sets and which is discussed later. Briefly said, the *RMSE* between the fused product and the panchromatic image is computed for each band $k$ and the spatial *ERGAS* is the mean of these *RMSEs*. We do not recommend the use of this distance because it cannot be considered that the panchromatic image is a reference for fused images. This distance should not be used.

## 2.4 *Category 4 "correlation between high frequencies"*

The category 4 is similar to category 3 because it evaluates similarities between spatial details. The difference lies in the fact that prior to the calculation of a distance, a high-pass filter is applied in order to extract an image of high frequencies. The distances are computed on these resulting images.

Zhou *et al*. (1998) and Li (2000) apply a Laplacian kernel to the images and then compute the correlation coefficient between the resulting images. This quantity is also called spatial correlation coefficient *sCC*. However, the above-mentioned authors do not apply this distance correctly because they use the PAN as a reference. This drawback was corrected by Otazu *et al*. (2005). The ideal value is 1.

Eskicioglu & Fisher (1995), Li *et al*. (2001), Zheng *et al*. (2005), Nencini *et al*. (2006) compute the gradient at each pixel and then average the gradient over all pixels for respectively $B^*_k$ and $B_k$. The distance is here the difference between the averaged gradients. This difference may be expressed as a relative value by dividing by the average gradient of $B_k$. The ideal value is zero.

Alonso-Reyes *et al*. (2005) extract the first plane of wavelet coefficients for respectively $B^*_k$ and $B_k$ and make a correlogram. In the ideal case, the points lie along the main diagonal (y = x). A linear regression is applied on the points and the parameters of the straight line are compared to the ideal values: 1 for the slope and 0 for the intercept.

## 2.5 *Category 5 "local sharpness"*

The category 5 deals with the difference in local sharpness. Several features are selected which are elongated well-contrasted edges. These tools are based on optical characteristics on sensors. The point spread function (*PSF*) is the response of the sensor to an ideal point light impulse. It reveals how the system blurs the contrast of the image as a function of the spatial frequencies. *PSF* has a Gaussian shape. The blur parameter is defined as the standard deviation of the Gaussian function. Li (2000) computes the difference in blur parameter between $B^*_k$ and $B_k$.

The modulation transfer function (*MTF*) corresponds to the derivate of the *PSF*. The MTF can be estimated on any image by an edge approach and therefore gives the frequency content of this edge. This tool is use to quantify the edge quality in fused image compared to the reference one; a frequency curve is obtained for each image (Thomas & Wald 2006b). A L2-norm is used to characterize the distance between the curves and quantifies the difference in local sharpness.

As these distances deal with differences between fused images and reference images, the ideal value for each of these distances is zero.

## 2.6 *Category 6 "spectral vector"*

In the category 6, distances aim at measuring the discrepancies in spectra between the fused and reference images. Spectra are also called spectral vectors or state vectors or *N-tuplets* or spectral signatures. These distances quantify the divergence in spectral information which is of paramount importance in classification processes.

Wald *et al*. (1997) propose to count the number of coincident spectra. Ideally, this number should be equal to the number of spectra in the reference image. One may also focus on the most frequent spectra because of their large influence on classification process. Discrepancies may be quantified by counting the number of coincident spectra and the number of pixels bearing one of these spectra – i.e., the frequency of each spectrum.

One can compute the spectral angle between vectors of the fused and reference images. Nencini *et al*. (2006) take the absolute value of this spectral angle at each pixel. The spectral angle mapper (*SAM*) is this angle and is usually averaged over the whole image to yield a global measurement of spectral distortion. Ideally, the *SAM* should be equal to 0.

Thomas *et al*. (2005) compute at each pixel the difference between the norm of the fused spectral vector and the reference one. Then, two distances are highlighted: the relative bias of this image of difference and its standard deviation relative to the mean of the reference norm. Ideal values are zero. One may also compute at each pixel the resultant vector defined as the difference between the spectral vectors, and then the norm of this resultant vector. An image is formed from which are kept the mean and the standard deviation. Ideal values are zero.

## 2.7 *Category 7 "global distances"*

The category 7 is made of distances aiming at providing a single quantity synthesizing the quality of the fused data set. Munechika *et al*. (1993) sum the *RMSE* for each band *k*.

Wald *et al*. (1997) propose an average of these *RMSE*. This approach was refined later by Wald (2002) who proposes the index *RASE* (relative average spectral error) and then a further refinement called *ERGAS*. *ERGAS* is the French acronym for relative dimensionless global error in synthesis. This index is often used and has the advantage to be independent of several factors such as the resolution, number of bands, etc. Its ideal value is 0.

The quality index *Q4* is a generalization to four multispectral bands of the monomodal index *Q* discussed earlier. This value is obtained through the use of correlation coefficients between hypercomplex numbers representing spectral vectors (Alparone *et al*. 2004; Nencini *et al*. 2006). The ideal value is 0.

The index *Qw* is also a generalization of the index *Q*. It is the average of the *Q* index computed on each band (Otazu *et al*. 2005). Its ideal value is 0.

# 3 RELATIONS BETWEEN DISTANCES

Each category quantifies a particular aspect of the fused images with respect to the reference. We illustrate in this section the complementarity of certain distances and the redundancies of others. The redundancy permits to reduce the number of distances to consider. The complementarity shows the optimal number of distances to compute to obtain a good quantitative assessment of the quality.

This section is based on illustration, mathematical explanations and empirical analysis of case studies. Seventy-three sets of fused images were created. They originate from various satellites (Quickbird, Ikonos, SPOT). The spectral bands are in the visible and infrared domain. Images cover a very high diversity of possible landscapes comprising many different spectral objects of small size, such as urban areas or mountains and farms. The fusion process was arbitrarily selected to be the model M2 proposed by Ranchin & Wald (2000). Ratios between the high and low resolutions are 2 or 4.

For each case, a visual analysis of the set of fused images is performed and several distances are computed. Distances apply to monospectral or to multispectral images, distances selected among those frequently used in the literature. We compare the results given by each distance and visual analysis in order to see how much this distance is an effective indicator of the quality and how it complements other distances.

Other distances, called global distances, characterize the overall quality of the fused set. We explored them as well as several combinations of individual distances, forming quality budgets. For example, a quality budget may contain only one distance if it is a global one, but may consist of several ones, selected in various categories. This aims at obtaining a more effective indicator of quality.

Our first example deals with categories 1, 2 and 3. Figure 1a (left) exhibits a Quickbird image of a road interchange. The right image is the same image but rotated by $180^\circ$. The information content of these two images is identical. They have the same

Figure 1. Quickbird image of a road interchange (left). Right: the same image but rotated by 180°. Copyright Digital Globe 2002.

spectra as well as the same most frequent spectra. The bias and difference in variance or entropy are null. Since category 1 deals only with global measurements, the information at pixel level is not taken into account and both images are identical for this category of distances. These distances are independent on rotation. On the contrary, distances in categories 2 and 3 are very sensitive to a change in pixel location – as well as other categories, except 1. The correlation coefficient or the index $Q$ has a very low value; the standard deviation of the image of differences is very large. This illustrates the fact that distances from one category should not be employed alone to quantify quality.

Figure 2 is another illustration. This Ikonos image exhibits fields and forests surrounding the town of Hasselt in Belgium (left). On the right hand, is the same image but where grey levels have been multiplied by two. As it is insensitive to an affine translation, i.e. by a multiplication or addition by a scalar, the correlation coefficient between both images reaches its ideal value of 1. On the contrary, distances in category 1, e.g., bias or difference in variance, exhibit very large values, far from their ideal value. The standard deviation of the difference image is far lower than in the previous illustration. This low value shows that the standard deviation is less discriminating in this case. This illustrates again the complementarities of the various categories.

We have investigated the relationship between the correlation coefficient and the index $Q$. The particularity of the fusion method M2 used in this empirical study is that it produces fused images with the same mean than the reference images. The consequence is that the bias is 0. As the index $Q$ is a function of the bias and the correlation coefficient, we investigate how much $Q$ and the correlation coefficient are similar. For all cases, the index $Q$ was drawn as a function of the correlation coefficient. Figure 3 exhibits the correlograms obtained for each ratio: 2 and 4. An extremely high similitude appears between the two metrics, except for some outliers. Our conclusion of this study is that in the case of a fusion method which produces fused images with the same mean


Figure 2. Ikonos image of fields and forests (left). Same image but after a multiplication of grey levels by two. Copyright Space Imaging 2002.

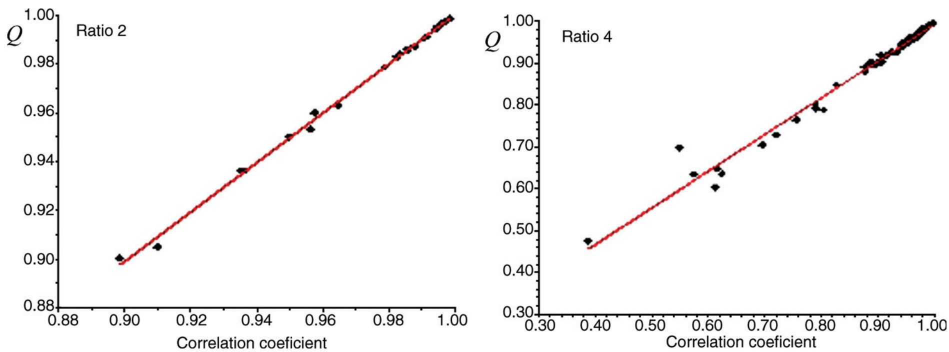


Figure 3. Correlograms of the index $Q$ and the correlation coefficient for two ratios of resolutions: 2 and 4. The regression straight lines are in red.

than the reference images, i.e., the case of most fusion methods based on multiscale analysis, and when the correlation coefficient is larger than 0.90, the correlation coefficient and the index $Q$ are equivalent. That is why we recommend to use the correlation coefficient in this case, because it is easier to implement.

The relationship between distances *PSF* and *MTF* in category 5 can be established on mathematical ground. It is not necessary to compute both of them; their results will be equivalent.

As for category 6 "spectral vector", the Spectral Angle Mapper (*SAM*), the difference between the norms of spectral vectors of the fused and reference data sets, denoted hereafter *diffNorms*, and the norm of the difference of the two spectral vectors, denoted hereafter *resultantVector*, are linked together by the means of the Al Kashi's theorem, better known under the name of generalized Pythagore Theorem. It says that:

$$\begin{aligned}||\mathit{resultantVector}(\mathrm{i},\mathrm{j})||^2 = ||\mathrm{B}(\mathrm{i},\mathrm{j})||^2 + ||\mathrm{B}^*(\mathrm{i},\mathrm{j})||^2 \\ -2||\mathrm{B}(\mathrm{i},\mathrm{j})||\ ||\mathrm{B}^*(\mathrm{i},\mathrm{j})||\cos(\mathit{SAM}(\mathrm{i},\mathrm{j}))\end{aligned}$$

If we develop the difference of norms into its expression as a function of the norms, it comes:

$$\begin{aligned}\mathit{diffNorms}^2(\mathrm{i},\mathrm{j}) &= (||\mathrm{B}(\mathrm{i},\mathrm{j})|| - ||B^*(i,j)||)^2 \\ &= ||\mathrm{B}(\mathrm{i},\mathrm{j})||^2 + ||\mathrm{B}^*(\mathrm{i},\mathrm{j})||^2 - 2||\mathrm{B}(\mathrm{i},\mathrm{j})||\ ||\mathrm{B}^*(\mathrm{i},\mathrm{j})|\end{aligned}$$

Finally, generalized Pythagore theorem can be rewritten as:

$$||\mathit{resultantVector}(\mathrm{i},\mathrm{j})||^2 = \mathit{diffNorms}^2(\mathrm{i},\mathrm{j}) + 2||\mathrm{B}(\mathrm{i},\mathrm{j})||\ ||\mathrm{B}^*(\mathrm{i},\mathrm{j})||(1-\cos(\mathit{SAM}(\mathrm{i},\mathrm{j})))$$

The *SAM* is becoming progressively a reference in the field of quality assessment of fused images. The two other norms complete information already given by this distance, with the risk to bring some redundancies. In order to avoid unnecessary computation, we advise to select only one of these three, with a preference for the *SAM*.

## 4 CONCLUSION

The interpretation of the observations gathered during this analysis of case studies results in a series of suggestions. Some were already reported and concern the redundancies of several distances, e.g., the correlation coefficient and the index *Q*, or the *SAM* and two spectral norms. The need for considering in a quality budget distances belonging to several categories appears clearly.

From our experience, we suggest the following list of distances. For each spectral band, compute:

- the bias in relative value, i.e., divided by the mean of the reference spectral image (category 1),
- the difference in variance in relative value, i.e., divided by the variance of the reference spectral image (category 1),
- the standard-deviation of the differences on a pixel basis, in relative value, i.e., divided by the mean of the reference spectral image (category 2),
- the correlation coefficient (category 3),
- the correlation between high frequencies, i.e., after a high-pass filtering (category 4),
- the local sharpness by the means of the *MTF* tool (category 5). However, one should be careful with this tool as it is very recent and a very little experience has been reported in the literature up to now.

For the whole data set, compute:

- the average *SAM* (category 6),
- the average relative difference between the norms of spectral vectors of the fused and reference data sets (category 6),
- the *ERGAS* (category 7).

This communication deals with distances and recommends several of them for quality assessment. This work completes the previous ones made in the framework of the EARSeL Special Interest Group, aiming at studying, proposing and recommending means for assessing the quality of fused images. Several relationships were found between distances. Our work is not exhaustive; other links between distances can be found or demonstrated.

Further work should focus on the criteria for stating whether a fused image is of good quality or not. Few efforts have been made on their definitions. An example is the empirical study made on ERGAS (Wald 2002) which stated that a fused image exhibiting an ERGAS better than three is of good quality. Similar works should be encouraged in order to achieve the common framework recommended by EARSeL to qualify fused images.

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